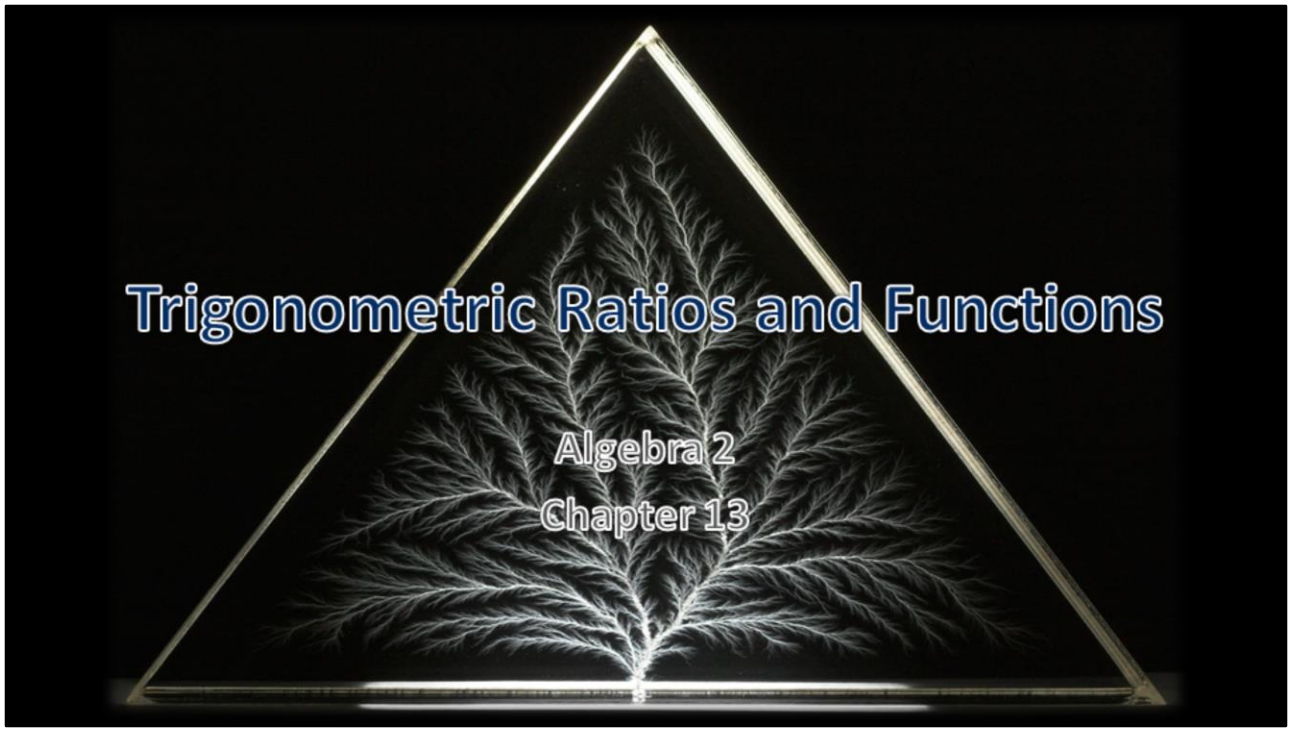

Algebra II 13

- This Slideshow was developed to accompany the textbook
  - *Larson Algebra 2*
  - *By Larson, R., Boswell, L., Kanold, T. D., & Stiff, L.*
  - *2011 Holt McDougal*
- Some examples and diagrams are taken from the textbook.

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13.1 Use Trigonometry with Right Triangles

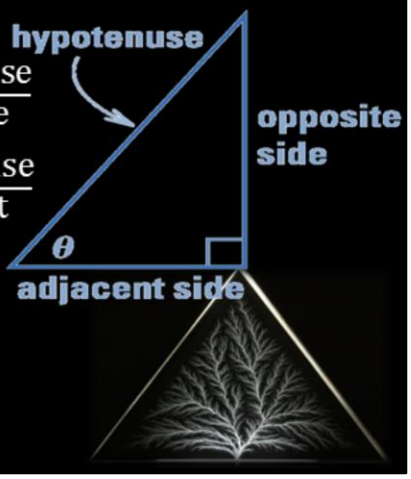
- If you have a right triangle, there are six ratios of sides that are always constant

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

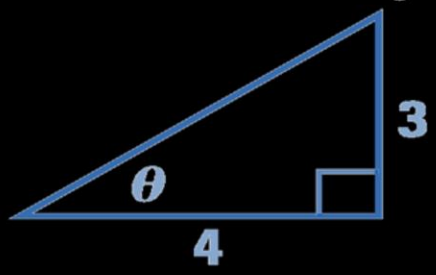
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

SOH
CAH
TOA



13.1 Use Trigonometry with Right Triangles

Evaluate the six trigonometric functions of the angle θ.



Use Pythagorean Theorem to find hypotenuse

$$3^2 + 4^2 = hyp^2$$
$$hyp = 5$$

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$

$\csc\theta = \frac{5}{3}$ $\sec\theta = \frac{5}{4}$ $\cot\theta = \frac{4}{3}$

Use Pythagorean Theorem to find adjacent

$$15^2 + adj^2 = 17^2$$
$$225 + adj^2 = 289$$
$$adj^2 = 64$$
$$adj = 8$$

$\sin\theta = \frac{15}{17}$ $\cos\theta = \frac{8}{17}$ $\tan\theta = \frac{15}{8}$

$\csc\theta = \frac{17}{15}$ $\sec\theta = \frac{17}{8}$ $\cot\theta = \frac{8}{15}$

Draw triangle and use pythagorean theorem to find opposite side

$$adj = 7$$
$$hyp = 10$$
$$7^2 + \text{opp}^2 = 10^2$$
$$opp^2 = 51$$
$$opp = \sqrt{51}$$

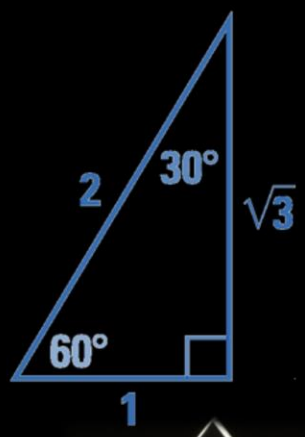
$$\sin\theta = \frac{\sqrt{51}}{10}$$

13.1 Use Trigonometry with Right Triangles

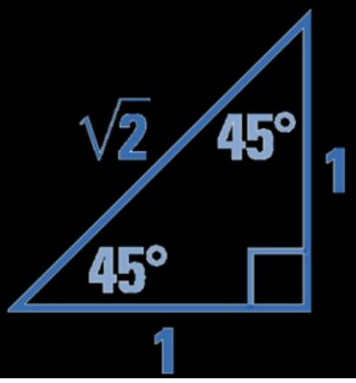
- In a right triangle, θ is an acute angle and $\cos\theta = \frac{7}{10}$. What is $\sin\theta$?

13.1 Use Trigonometry with Right Triangles

- Special Right Triangles
- 30° - 60° - 90°



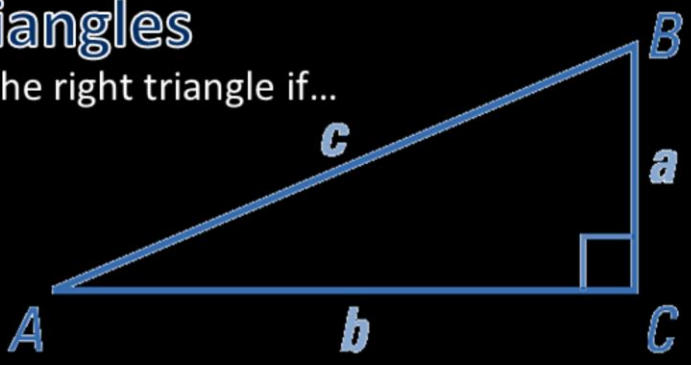
- 45° - 45° - 90°



These triangles can be used to find sin, cos, tan of 30, 60, and 45 degree angles exactly.

13.1 Use Trigonometry with Right Triangles

- Use the diagram to solve the right triangle if…
- B = 45°, c = 5
- B = 60°, a = 7
- A = 32°, b = 10



A = 90° - 45° = 45°

From special rt triangle, $\sin 45° = \frac{a}{5} = \frac{1}{\sqrt{2}} \rightarrow a = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

$\cos 45° = \frac{b}{5} = \frac{1}{\sqrt{2}} \rightarrow b = \frac{5\sqrt{2}}{2}$

A = 90° - 60° = 30°

From special rt triangle; $\tan 60° = \frac{b}{7} = \frac{\sqrt{3}}{1} \rightarrow b = 7\sqrt{3}$

$\cos 60° = \frac{7}{c} = \frac{1}{2} \rightarrow c = 14$

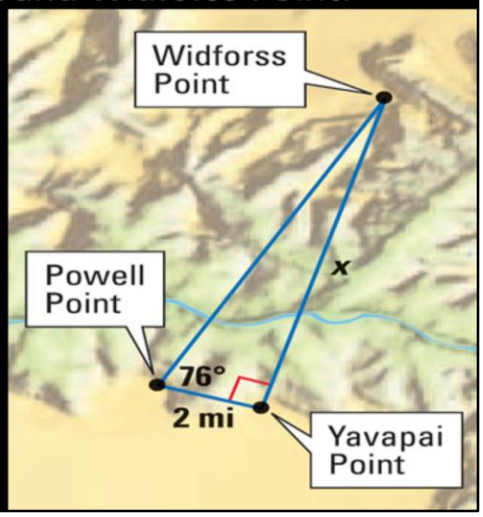
B = 90° - 32° = 58°

$\tan 32° = \frac{a}{10} \rightarrow a = 10 \tan 32° \approx 6.25$

$\cos 32° = \frac{10}{c} \rightarrow c \cdot \cos 32° = 10 \rightarrow c = \frac{10}{\cos 32°} \approx 11.79$

13.1 Use Trigonometry with Right Triangles

- Find the distance between Powell Point and Widforss Point.



- *856 #1-27 odd, 31, 33, 37 + 3 = 20*

$$\cos 76° = \frac{2 \text{ mi}}{y}$$

$$y \cdot \cos 76° = 2 \text{ mi}$$

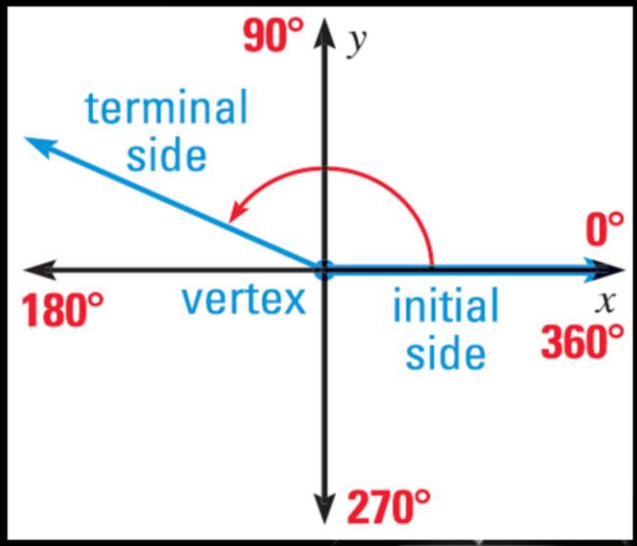
$$y = \frac{2 \text{ mi}}{\cos 76°} \approx 8.27 \text{ mi}$$

Quiz

- 13.1 Homework Quiz

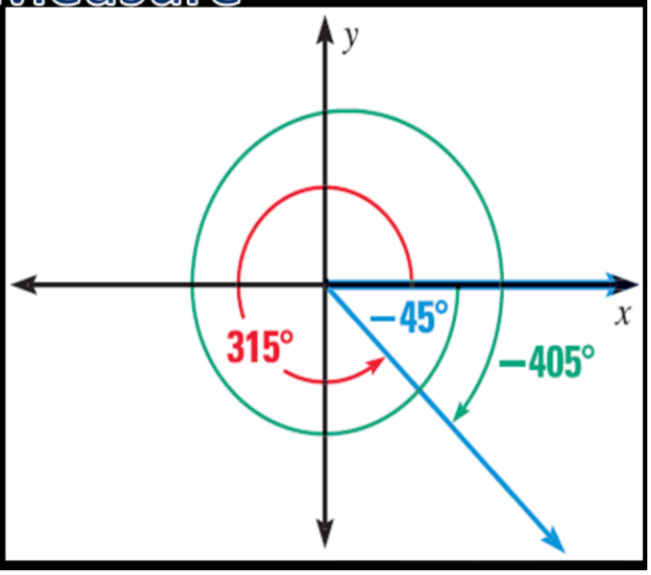
13.2 Define General Angles and Use Radian Measure

- Angles in Standard Position
  - Vertex on origin
  - Initial Side on positive x-axis
  - Measured counterclockwise



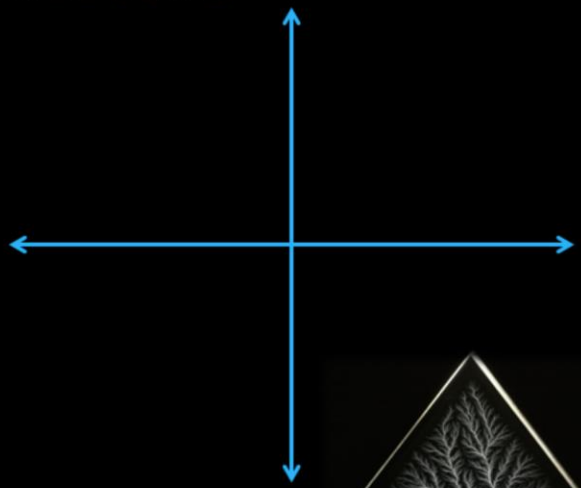
13.2 Define General Angles and Use Radian Measure

- Coterminal Angles
  - Different angles (measures) that have the same terminal side
  - Found by adding or subtracting multiples of 360°



13.2 Define General Angles and Use Radian Measure

- Draw an angle with the given measure in standard position. Then find one positive coterminal angle and one negative coterminal angle.
- 65°
- 300°

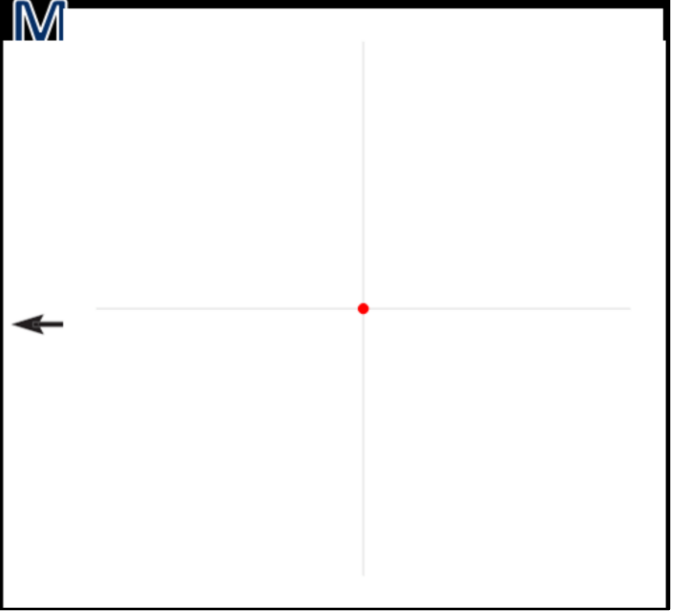


65°+360°=425°; 65°-360°=-295°

300°+360°=660°; 300°-360°=-60°

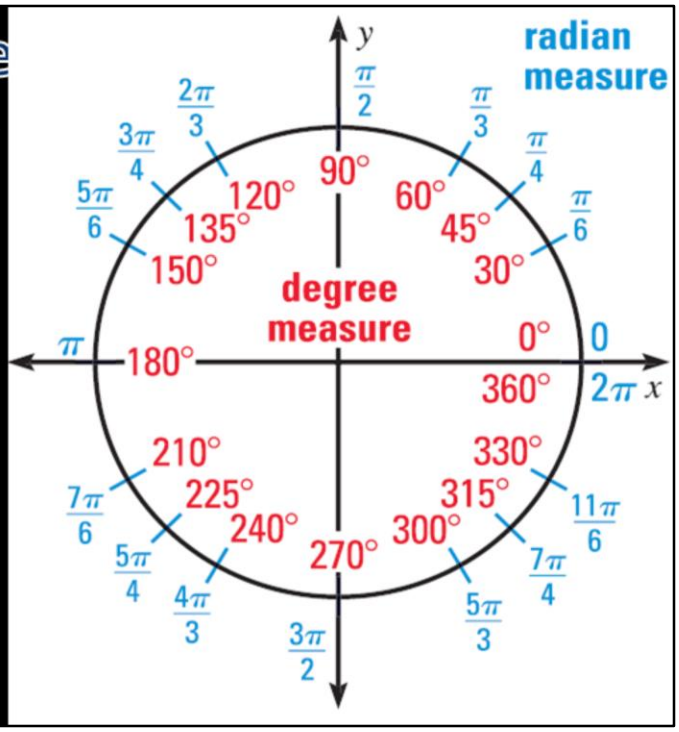
13.2 Define General Angles and Use Radian M

- Radian measure
  - Another unit to measure angles
  - 1 radian is the angle when the arc length = the radius
  - There are 2π radians in a circle



13.2 Define Gene Radian

- To convert between degrees and radians use fact that
  - 180° = π
- Special angles



Probably only need to memorize 1st quadrant

13.2 Define General Angles and Use Radian Measure

- Convert the degree measure to radians, or the radian measure to degrees.
- 135°
- -50°
- $\frac{5\pi}{4}$
- $\frac{\pi}{10}$

$$135°\left(\frac{\pi}{180}°\right) = \frac{3\pi}{4}$$

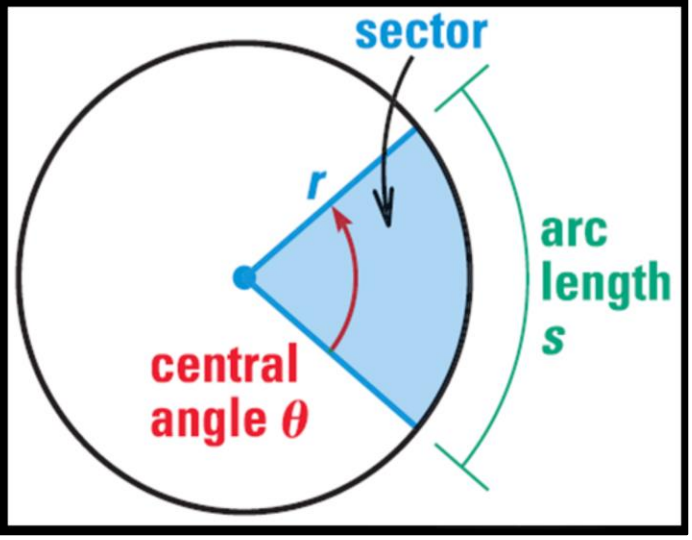
$$-50°\left(\frac{\pi}{180}°\right) = -\frac{5\pi}{18}$$

$$\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$$

$$\frac{\pi}{10}\left(\frac{180°}{\pi}\right) = 18°$$

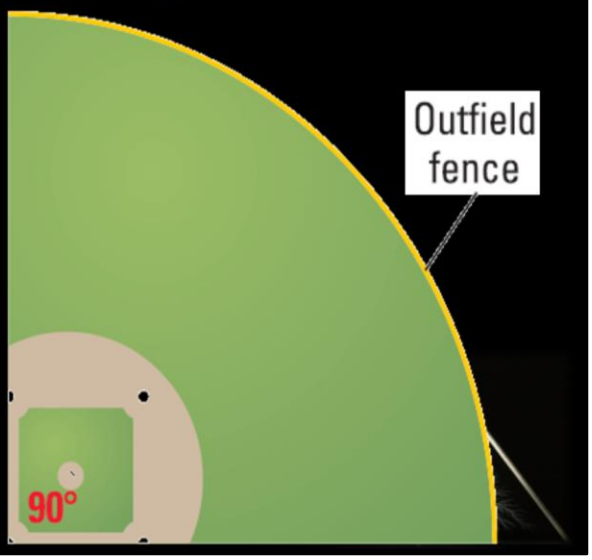
13.2 Define General Angles and Use Radian Measure

- Sector
  - Slice of a circle
- Arc Length
  - $s = r\theta$
  - θ must be in radians!
- Area of Sector
  - $A = \frac{1}{2}r^2\theta$
  - θ must be in radians!



13.2 Define General Angles and Use Radian Measure

- Find the length of the outfield fence if it is 220 ft from home plate.
- Find the area of the baseball field.
- *862 #3-51 odd + 0 = 25*



$$\theta = \frac{\pi}{2}$$

$$s = r\theta$$

$$s = 220\left(\frac{\pi}{2}\right) = 110\pi \approx 346$$

$$A = \frac{1}{2}r^2\theta$$

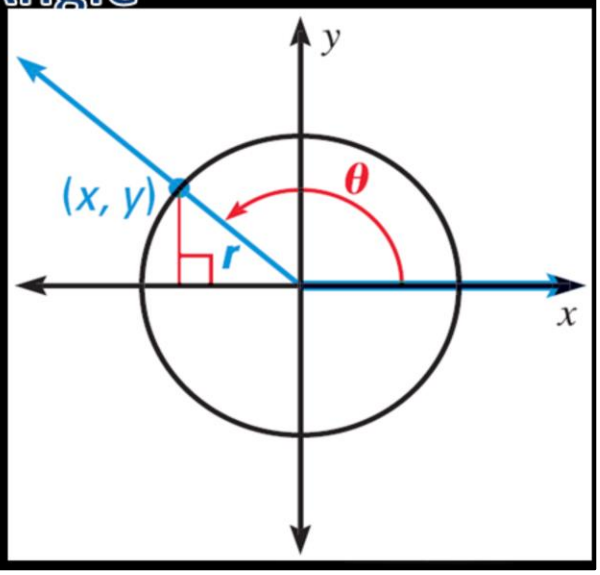
$$A = \frac{1}{2}(220)^2\left(\frac{\pi}{2}\right) = 12100\pi \approx 38013$$

Quiz

- 13.2 Homework Quiz

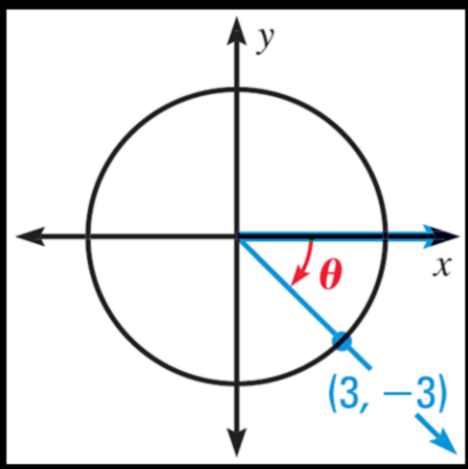
13.3 Evaluate Trigonometric Functions of Any Angle

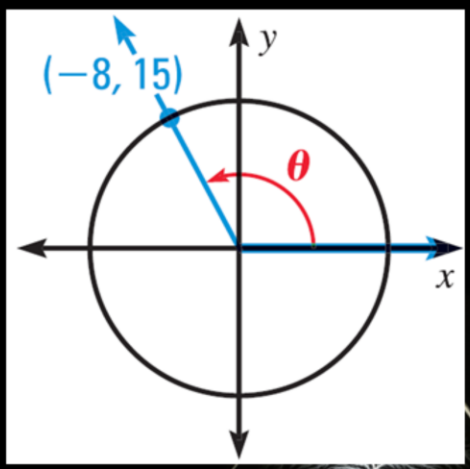
- Think of a point on the terminal side of an angle
- You can draw a right triangle with the x-axis
- $\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$
- $\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$
- $\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$
- Unit Circle
  - r = 1



13.3 Evaluate Trigonometric Functions of Any Angle

Evaluate the six trigonometric functions of θ.





$$r^2 = x^2 + y^2 = 3^2 + (-3)^2$$
$$r = \sqrt{18} = 3\sqrt{2}$$

$\sin\theta = -\frac{3}{3\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\cos\theta = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\tan\theta = -\frac{3}{3} = -1$

$\csc\theta = -\sqrt{2}$ $\sec\theta = \sqrt{2}$ $\cot\theta = -1$

$$r^2 = (-8)^2 + 15^2 = 289$$
$$r = 17$$

$\sin\theta = \frac{15}{17}$ $\cos\theta = -\frac{8}{17}$ $\tan\theta = -\frac{15}{8}$

$\csc\theta = \frac{17}{15}$ $\sec\theta = -\frac{17}{8}$ $\cot\theta = -\frac{8}{15}$

$\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$

$\csc\theta = und$ $\sec\theta = -1$ $\cot\theta = und$

13.3 Evaluate Trigonometric Functions of Any Angle

- Evaluate the six trigonometric functions of θ.
- θ = 180°

$$r^2 = x^2 + y^2 = 3^2 + (-3)^2$$
$$r = \sqrt{18} = 3\sqrt{2}$$

$\sin\theta = -\frac{3}{3\sqrt{2}} = -\frac{\sqrt{2}}{2}$ $\cos\theta = \frac{3}{3\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\tan\theta = -\frac{3}{3} = -1$

$\csc\theta = -\sqrt{2}$ $\sec\theta = \sqrt{2}$ $\cot\theta = -1$

$$r^2 = (-8)^2 + 15^2 = 289$$
$$r = 17$$

$\sin\theta = \frac{15}{17}$ $\cos\theta = -\frac{8}{17}$ $\tan\theta = -\frac{15}{8}$

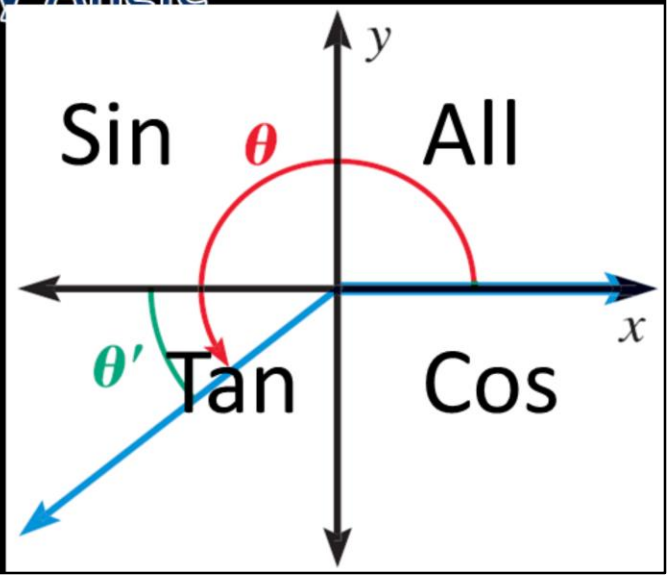
$\csc\theta = \frac{17}{15}$ $\sec\theta = -\frac{17}{8}$ $\cot\theta = -\frac{8}{15}$

$\sin\theta = 0$ $\cos\theta = -1$ $\tan\theta = 0$

$\csc\theta = und$ $\sec\theta = -1$ $\cot\theta = und$

13.3 Evaluate Trigonometric Functions of Any Angle

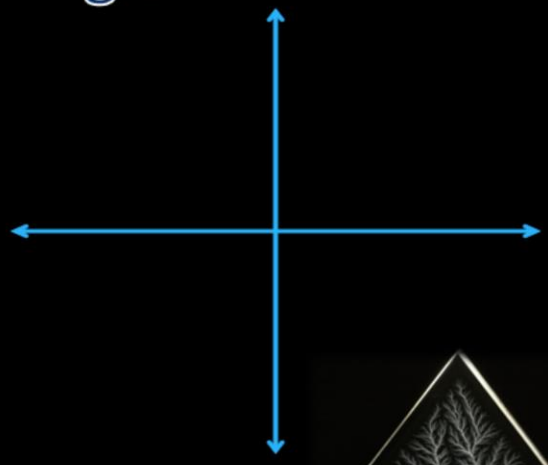
- Reference Angle
  - Angle between terminal side and x-axis
  - Has the same values for trig functions as 1st quadrant angles
  - You just have to add the negative signs



This gives what is positive.
The reciprocal functions are the same (csc is with sin, etc.)
Way to remember "All Students Take Calculus"

13.3 Evaluate Trigonometric Functions of Any Angle

- Sketch the angle. Then find its reference angle.
- $150°$
- $-\frac{7\pi}{9}$



- Evaluate cos(-60°) without a calculator

$$180° - 150° = 30°$$

$$\pi - \frac{7\pi}{9} = \frac{2\pi}{9}$$

Reference angle is 60°
In quadrant IV

$$\cos(-60°) = \frac{1}{2}$$

13.3 Evaluate Trigonometric Functions of Any Angle

- Estimate the horizontal distance traveled by a Red Kangaroo who jumps at an angle of 8° and with an initial speed of 53 feet per second (35 mph).

- *870 #3-37 odd + 2 = 20*

$$d = \frac{v^2}{32}\sin 2\theta$$

$$d = \frac{53^2}{32}\sin\big(2(8°)\big) = 24.2\ ft$$

Quiz

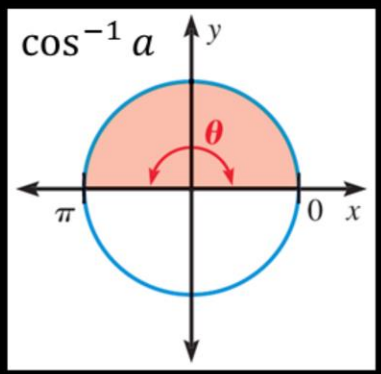
- 13.3 Homework Quiz

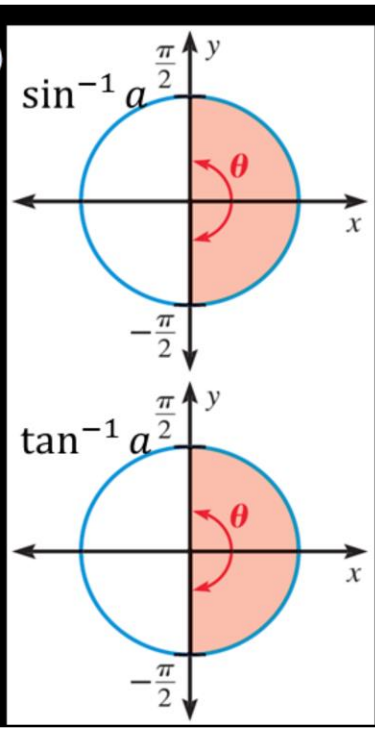
13.4 Evaluate Inverse Trigonometric Functions

- Find an angle whose tangent = 1
- There are many
  - $\frac{\pi}{4}, \frac{5\pi}{4}, -\frac{3\pi}{4}$, etc.
- In order to find angles given sides (or x and y) we have to define the functions carefully

13.4 Evaluate Inverse Trigo Functions

- Inverse Trig Functions
- $\sin^{-1} a = \theta$
  - $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
- $\cos^{-1} a = \theta$
  - $0 \leq \theta \leq \pi$
- $\tan^{-1} a = \theta$
  - $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$





13.4 Evaluate Inverse Trigonometric Functions

- Evaluate the expression in both radians and degrees.
- $\sin^{-1}\frac{\sqrt{2}}{2}$
- $\cos^{-1}\frac{1}{2}$
- $\tan^{-1}-1$

$$\sin^{-1}\frac{\sqrt{2}}{2} = 45° \text{ or } \frac{\pi}{4}$$

$$\cos^{-1}\frac{1}{2} = 60° \text{ or } \frac{\pi}{3}$$

$$\tan^{-1}-1 = -45° \text{ or } -\frac{\pi}{4}$$

13.4 Evaluate Inverse Trigonometric Functions

- Solve the equation for θ
- $\cos\theta = 0.4; 270° < \theta < 360°$
- $\tan\theta = 4.7; 180° < \theta < 270°$
- $\sin\theta = 0.62; 90° < \theta < 180°$

$$\cos\theta = 0.4$$
$$\theta = \cos^{-1} 0.4 = 66.4°$$

Quadrant IV with reference angle 66.4°

$$\theta = 360° - 66.4° = 293.6°$$

$$\tan\theta = 4.7$$
$$\theta = \tan^{-1} 4.7 = 78.0°$$

Quadrant III with reference angle 78.0°

$$180° + 78.0° = 258°$$

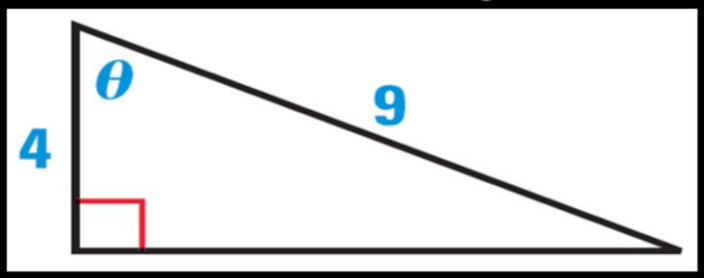
$$\sin\theta = 0.62$$
$$\theta = \sin^{-1} 0.62 = 38.3°$$

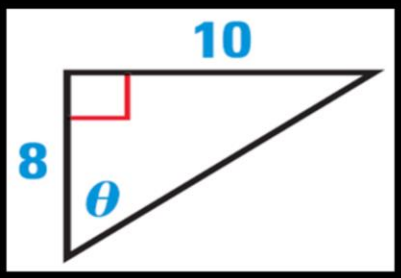
Quadrant II with reference angle 38.3°

$$180° - 38.3° = 141.7°$$

13.4 Evaluate Inverse Trigonometric Functions

- Find the measure of angle θ.





- *878 #1-29 odd, 35, 37 + 3 = 20*

$$\cos\theta = \frac{4}{9}$$

$$\theta = \cos^{-1}\frac{4}{9} = 63.6°$$

$$\tan\theta = \frac{10}{8}$$

$$\theta = \tan^{-1}\frac{10}{8} = 51.3°$$

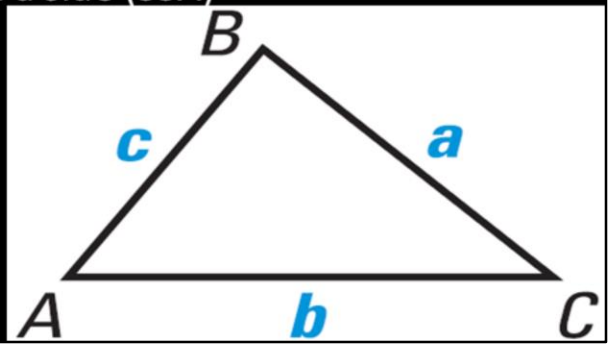
Quiz

- 13.4 Homework Quiz

13.5 Apply the Law of Sines

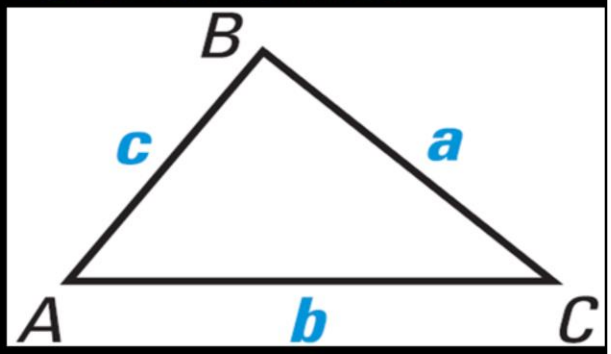
- In lesson 13.1 we solved right triangles
- In this lesson we will solve any triangle if we know
  - 2 Angles and 1 Side (AAS or ASA)
  - 2 Sides and 1 Angle opposite a side (SSA)

- Law of Sines
  - $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$



13.5 Apply the Law of Sines

- Solve ΔABC if…
  - A = 51°, B = 44°, c = 11



$$C = 180° - 51° - 44° = 85°$$

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 85°}{11} = \frac{\sin 51°}{a}$$

$$a \cdot \sin 85° = 11 \cdot \sin 51°$$

$$a = \frac{11 \cdot \sin 51°}{\sin 85°} \approx 8.58$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

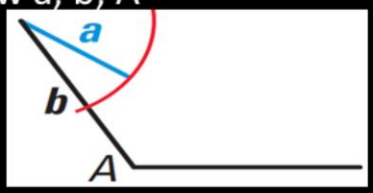
$$\frac{\sin 85°}{11} = \frac{\sin 44°}{b}$$

$$b \cdot \sin 85° = 11 \cdot \sin 44°$$

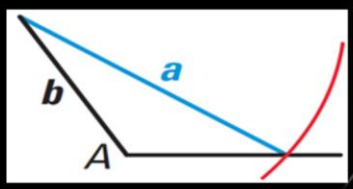
$$b = \frac{11 \cdot \sin 44°}{\sin 85°} \approx 7.67$$

13.5 Apply the Law of Sines

- Indeterminant Case (SSA)
  - Maybe no triangle, one triangle, or two triangles
- In these examples, you know a, b, A
- If A > 90° and…
  - a ≤ b → no triangle

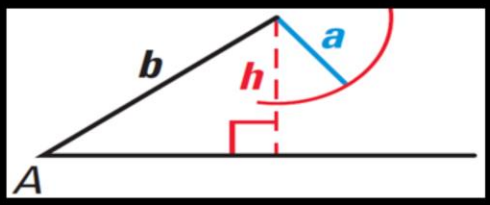


  - a > b → 1 triangle

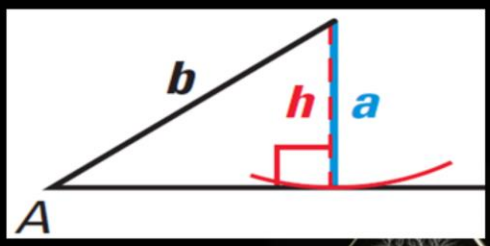


13.5 Apply the Law of Sines

- A < 90° and… (h = b sin A)
  - h > a → no triangle

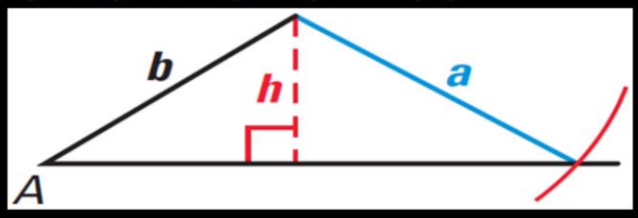


  - h = a → one triangle

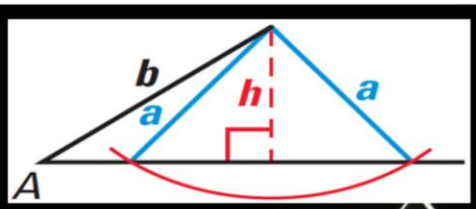


13.5 Apply the Law of Sines

- a ≥ b → one triangle



- h < a < b → two triangles



13.5 Apply the Law of Sines

- Solve ΔABC
  - A = 122°, a = 18, b = 12
  - A = 36°, a = 9, b = 12

a > b → one triangle

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 122°}{18} = \frac{\sin B}{12}$$
$$18 \sin B = 12 \sin 122°$$
$$\sin B = \frac{12 \sin 122°}{18} \approx 0.5654$$
$$B = \sin^{-1} 0.5654 \approx 34.4°$$
$$C = 180° - 34.4° - 122° = 23.6°$$
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$
$$\frac{\sin 122°}{18} = \frac{\sin 23.6°}{c}$$
$$c \cdot \sin 122° = 18 \sin 23.6°$$
$$c = \frac{18 \sin 23.6°}{\sin 122°} \approx 8.50$$

h = b sin A = 12 sin 36° = 7.05; h < a < b → two solutions

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 36°}{9} = \frac{\sin B}{12}$$
$$9 \sin B = 12 \sin 36°$$
$$\sin B = \frac{12 \sin 36°}{9} \approx 0.7837$$
$$B = \sin^{-1} 0.7837 \approx 51.6°$$

To angles for B and they are supplementary

$B = 51.6°$ $B' = 180° - 51.6° = 128.4°$

$C = 180° - 51.6° - 36° = 92.4°$ $C' = 180° - 128.4° - 36° = 15.6°$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

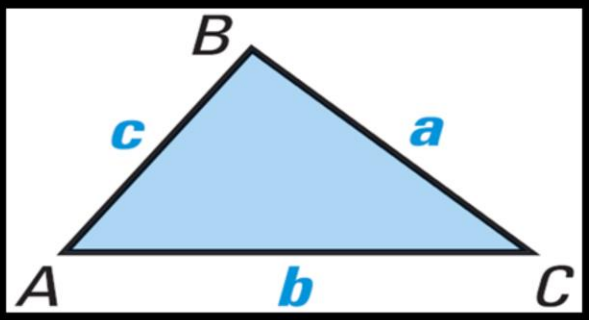
$\frac{\sin 36°}{9} = \frac{\sin 92.4°}{c}$ $\frac{\sin 36°}{9} = \frac{\sin 15.6°}{c}$

$c \sin 36° = 9 \sin 92.4°$ $c \sin 36° = 9 \sin 15.6°$

$c = \frac{9 \sin 92.4°}{\sin 36°} \approx 15.3$ $c = \frac{9 \sin 15.6°}{\sin 36°} \approx 4.1$

13.5 Apply the Law of Sines

- Area of Triangle
  - $Area = \frac{1}{2}bh$
  - $h = c \sin A$
  - $Area = \frac{1}{2}bc \sin A$
- Find the area of ΔABC with…
  - a = 10, b = 14, C = 46°



- *886 #1-25 odd, 29-39 odd, 43, 45 + 4 = 25*

$$Area = \frac{1}{2}ab \sin C$$

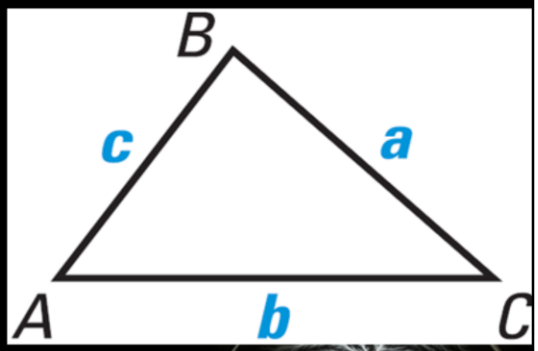
$$Area = \frac{1}{2}(10)(14) \sin 46° \approx 50.4$$

Quiz

- 13.5 Homework Quiz

13.6 Apply the Law of Cosines

- When you need to solve a triangle and can't use Law of Sines, use Law of Cosines
  - 2 Sides and Included angle (SAS)
  - 3 Sides (SSS)
- Law of Cosines
  - $a^2 = b^2 + c^2 - 2bc\cos A$
  - $b^2 = a^2 + c^2 - 2ac\cos B$
  - $c^2 = a^2 + b^2 - 2ab\cos C$



You can use the Law of Cosines, then Law of Sines as long as you don't create the SSA case.

13.6 Apply the Law of Cosines

- Solve ΔABC if…
  - a = 8, c = 10, B = 48°
  - a = 14, b = 16, c = 9

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$b^2 = 8^2 + 10^2 - 2(8)(10)\cos 48° = 56.94$$
$$b = 7.55$$
$$a^2 = b^2 + c^2 - 2ab\cos A$$
$$8^2 = 7.55^2 + 10^2 - 2 \cdot 7.55 \cdot 10\cos A$$
$$64 = 57.00 + 100 - 151\cos A$$
$$-93 = -151\cos A$$
$$0.61589404 = \cos A$$
$$A = \cos^{-1} 0.61589404 \approx 52.0°$$
$$C = 180° - 48° - 52.0° = 80°$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$14^2 = 16^2 + 9^2 - 2(16)(9)\cos A$$
$$196 = 256 + 81 - 288\cos A$$
$$-141 = -288\cos A$$
$$0.4896 = \cos A$$
$$A = \cos^{-1} 0.4896 \approx 60.7°$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$16^2 = 14^2 + 9^2 - 2 \cdot 14 \cdot 9\cos B$$
$$256 = 196 + 81 - 252\cos B$$
$$-21 = -252\cos B$$
$$0.08333333333 = \cos B$$

$$B = \cos^1 0.0833333 \approx 85.2°$$
$$C = 180° - 60.7° - 85.2° = 34.1°$$

13.6 Apply the Law of Cosines

- Heron's Area Formula
  - $Area = \sqrt{s(s-a)(s-b)(s-c)}$
  - Where $s = \frac{1}{2}(a+b+c)$
- Find the area of ΔABC

- *892 #3-31 odd, 37-45 odd + 0 = 20*

$$s = \frac{1}{2}(a+b+c)$$

$$s = \frac{1}{2}(8+11+5) = 12$$

$$Area = \sqrt{s(s-a)(s-b)(s-c)}$$

$$Area = \sqrt{12(12-8)(12-11)(12-5)} = \sqrt{336} = 18.3$$

Quiz

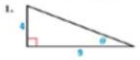
- 13.6 Homework Quiz

13.Review

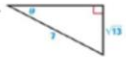
- 901 #choose 20 = 20

13 CHAPTER TEST

Evaluate the six trigonometric functions of the angle θ.

1. 

2. 

3. 

Convert the degree measure to radians or the radian measure to degrees.

4. $260°$ 5. $-50°$ 6. $\frac{4\pi}{5}$ 7. $\frac{8\pi}{3}$

Evaluate the function without using a calculator.

8. $\tan 150°$ 9. $\sec(-480°)$ 10. $\sin\left(-\frac{5\pi}{3}\right)$ 11. $\cos\frac{11\pi}{6}$

Evaluate the expression in both radians and degrees without using a calculator.

12. $\cos^{-1} 1$ 13. $\tan^{-1}\sqrt{3}$ 14. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 15. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

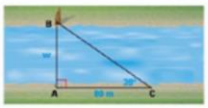
Solve $\triangle ABC$. (*Hint:* Some of the "triangles" may have no solution and some may have two solutions.)

16. $A = 47°, C = 32°, c = 12$
17. $a = 24, b = 12, c = 17$
18. $B = 63°, a = 11, b = 8$
19. $C = 101°, a = 23, b = 19$
20. $a = 24, b = 30, c = 21$
21. $A = 26°, B = 77°, c = 50$

Find the area of $\triangle ABC$.

22. $A = 81°, b = 16, c = 18$
23. $a = 8, b = 6, c = 7$
24. $a = 25, b = 24, c = 38$
25. $C = 111°, a = 7, b = 13$
26. $a = 16, b = 33, c = 24$
27. $B = 61°, a = 12, c = 18$

28. **SURVEYING** To measure the width of a river, you plant a stake at point A on one side of the riverbank, directly across from a tree stump at point B on the other side of the riverbank. From point A, you walk 80 meters along the riverbank to point C. You find the measure of angle C to be 39°. What is the width w of the river?



29. **CONSTRUCTION** A crane has a 200 foot arm with a lower end that is 5 feet off the ground. The arm has to reach to the top of a building that is 160 feet high. At what angle θ should the arm be set?

30. **NAVIGATION** A boat travels 40 miles due west before turning 20° and traveling an additional 25 miles. How far is the boat from its point of departure?